8.5 Volume of a Cone

Cone-shaped containers are used in a variety of professions, such as environmental studies, agriculture, and culinary arts.

In this section, you will develop a formula for the volume of any cone.

Investigate

Tools

- empty cylindrical can
- construction paper
- scissors
- tape
- sand, rice, or another suitable material

How can you model the volume of a cone?

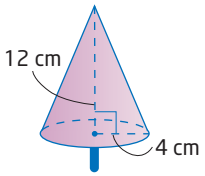
Work with a partner.

1. Measure the radius and height of the can.
2. Construct a cone with the same base radius and height as the can.
   a) Use the radius and height to calculate the slant height of the cone.
   b) Construct a circle with a radius equal to the slant height you determined. Make a cut along a radius so that the circle can be formed into a cone.
   c) The cone's circumference should fit the circumference of the can. Tape the seam to form a cone.

3. Fill the cone with sand, rice, or another suitable material. Empty the rice into the can. Repeat until the can is full. How many cones of material does it take to fill the can?
4. a) **Reflect** What conclusion can you draw about the relationship between the volume of a cone and the volume of a cylinder with the same height and radius?
   b) You know the formula for the volume of a cylinder. Use your conclusion from part a) to write a formula for the volume, V, of a cone in terms of the radius, r, of the base and the height, h.

Example 1 Volume of a Frozen Yogurt Treat

Tracy makes her own frozen yogurt treats in cone-shaped paper cups. Determine the volume of the frozen yogurt treat shown, to the nearest cubic centimetre.



Solution

The volume of a cone is one third the volume of the cylinder with the same base and height.

$$V_{\text{cone}} = \frac{1}{3}(\text{volume of a cylinder})$$

$$= \frac{1}{3}\pi r^2 h$$

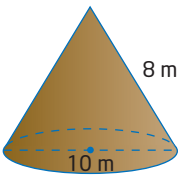
$$= \frac{1}{3}\pi(4)^2(12)$$ Estimate: $\frac{1}{3}(3)(4^2)(12) = (16)(12)$

$$\doteq 201$$ $= 192$

The volume of the frozen yogurt treat is approximately 201 cm^3.

Example 2 Volume of a Sand Pile

A conical pile of sand has a base diameter of 10 m and a slant height of 8 m. Determine the volume of the sand in the pile, to the nearest cubic metre.

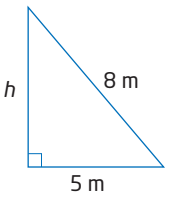


Solution

Since the diameter of the base is 10 m, the radius is 5 m.

To determine the volume of the cone, you need to know the height.

Apply the Pythagorean theorem.



$$s^2 = h^2 + r^2$$
$$8^2 = h^2 + 5^2$$
$$64 = h^2 + 25$$
$$h^2 = 64 - 25$$
$$h^2 = 39$$
$$h = \sqrt{39}$$
$$h \doteq 6.2$$

The height of the cone is approximately 6.2 m.

Now, determine the volume.

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi(5)^2(6.2)$$ Estimate: $\frac{1}{3}(3)(5^2)(6) = (25)(6)$ $= 150$

$$\doteq 162$$ C 1 ÷ 3 × π × 5 x^2 × 6.2 =

The volume of the sand in the pile is approximately 162 m^3.

Example 3 Find the Height of a Container

A fountain firework is packaged in a conical container. Its volume is 210 cm^3. Its diameter is 8 cm. What is the height of the fountain firework, to the nearest tenth of a centimetre?

Solution

Substitute the given values into the formula for the volume of a cone.

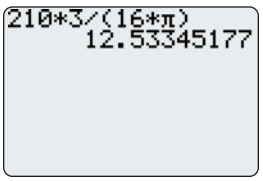
$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h$$

$$210 = \frac{1}{3}\pi(4)^2 h$$ Since the diameter is 8 cm, the radius is 4 cm.

$$210 = \frac{16\pi}{3}h$$

$$210 \times \frac{3}{16\pi} = h$$ To isolate h, I'll divide both sides by 16π and multiply both sides by 3.

$$h \doteq 12.5$$



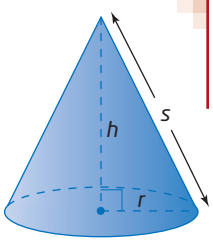
The height of the conical firework is approximately 12.5 cm.

Key Concepts

- The volume of a cone is one third the volume of a cylinder with the same base radius and height:

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h$$

- If you know the slant height, s, and base radius, r, of a cone, you can use the Pythagorean theorem to determine the height, h, of the cone.



Communicate Your Understanding

C1 A cylindrical container and a conical container have the same radius and height. How are their volumes related? How could you illustrate this relationship for a friend?

C2 Suppose the height of a cone is doubled. How will this affect the volume?

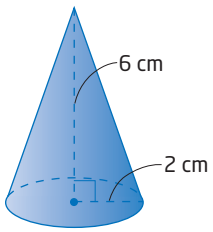
C3 Suppose the radius of a cone is doubled. How will this affect the volume?

Practise

For help with question 1, see Example 1.

1. Determine the volume of each cone. Round to the nearest cubic unit.

a)



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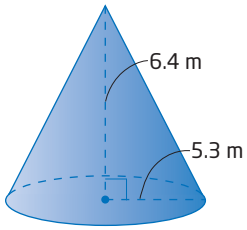

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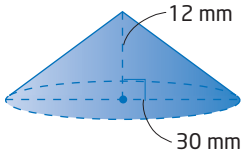

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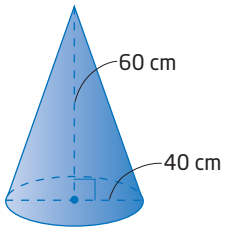
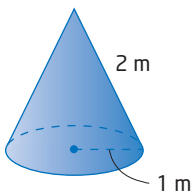


For help with questions 2 and 3, see Example 2.

2. Determine the volume of each cone. Round to the nearest cubic unit.

a)



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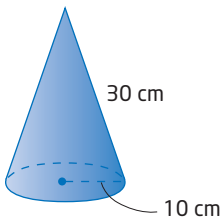
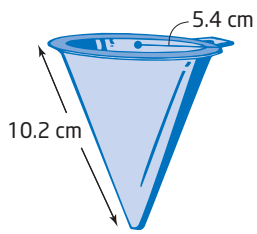


3. Wesley uses a cone-shaped funnel to put oil in a car engine. The funnel has a radius of 5.4 cm and a slant height of 10.2 cm. How much oil can the funnel hold, to the nearest tenth of a cubic centimetre?



For help with question 4, see Example 3.

4. A cone-shaped paper cup has a volume of 67 cm^3 and a diameter of 6 cm. What is the height of the paper cup, to the nearest tenth of a centimetre?

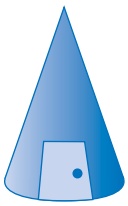
Connect and Apply

5. A cone just fits inside a cylinder with volume 300 cm^3. What is the volume of the cone?

6. Create a problem involving the volume of a cone. Solve it. Exchange your problem with a classmate.

7. A cone has a volume of 150 cm^3. What is the volume of a cylinder that just holds the cone?

8. A cone-shaped storage unit at a highway maintenance depot holds 4000 m^3 of sand. The unit has a base radius of 15 m.



a) Estimate the height of the storage unit.

b) Calculate the height.

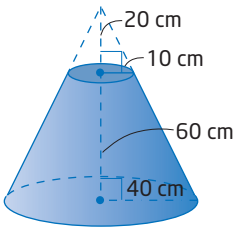
c) How close was your estimate?

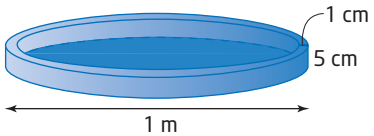
9. A cone has a height of 4 cm and a base radius of 3 cm. Another cone has a height of 3 cm and a base radius of 4 cm.

a) Predict which cone has the greater volume. Explain your prediction.

b) Calculate the volume of each cone, to the nearest cubic centimetre. Was your prediction correct?

10. **Chapter Problem** Refer to question 11 in Section 8.4. Determine the volume of concrete in Emily's birdbath. Round your answer to the nearest cubic centimetre.





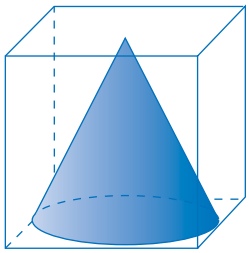
11. **a)** Express the height of a cone in terms of its volume and its radius.

b) If a cone holds 1 L and its radius is 4 cm, what is its height? Round your answer to the nearest tenth of a centimetre.

12. A cone-shaped funnel holds 120 mL of water. If the height of the funnel is 15 cm, determine the radius, rounded to the nearest tenth of a centimetre.

Extend

13. A cone just fits inside a cube with sides that measure 10 cm.



a) What are the dimensions of the largest cone that fits inside this box?

b) Estimate the ratio of the volume of the cone to the volume of the cube.

c) Calculate the volume of the cone, to the nearest cubic centimetre.

d) Calculate the ratio in part b).

e) How close was your estimate?

14. A cone has a height equal to its diameter. If the volume of the cone is 200 m^3, determine the height of the cone, to the nearest tenth of a metre.

15. **Use Technology** Use a graphing calculator, *The Geometer's Sketchpad*®, or a spreadsheet to investigate how the volume of a cone is affected when its radius is constant and its height changes.

16. **Use Technology** A cone has a height of 20 cm.

a) Write an algebraic model for the volume of the cone in terms of the radius.

b) Choose a tool for graphing. Graph the volume of the cone versus the radius.

c) Describe the relationship using mathematical terms.

17. **Math Contest** A cube has side length 6 cm. A square-based pyramid has side length 6 cm and height 12 cm. A cone has diameter 6 cm and height 12 cm. A cylinder has diameter 6 cm and height 6 cm. Order the figures from the least to the greatest volume. Select the correct order.

A cube, pyramid, cone, cylinder

B cylinder, cube, cone, pyramid

C cube, cone, cylinder, pyramid

D cone, pyramid, cylinder, cube

E pyramid, cone, cylinder, cube